NAG Toolbox for MATLAB

f02ga

1 Purpose

f02ga computes all the eigenvalues, and optionally the Schur factorization, of a complex general matrix.

2 Syntax

```
[a, w, z, ifail] = f02ga(job, a, 'n', n)
```

3 Description

f02ga computes all the eigenvalues, and optionally the Schur form or the complete Schur factorization, of a complex general matrix A:

$$A = ZTZ^{\mathrm{H}},$$

where T is an upper triangular matrix, and Z is a unitary matrix. T is called the *Schur form* of A, and the columns of Z are called the *Schur vectors*.

If it is desired to order the Schur factorization so that specified eigenvalues occur in the leading positions on the diagonal of T, then this function may be followed by a call of f08qu. Other reorderings may be achieved by calls to f08qt.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **job – string**

Indicates whether the Schur form and Schur vectors are to be computed.

job = 'N'

Only eigenvalues are computed.

job = 'S'

Eigenvalues and the Schur form T are computed.

job = 'V'

Eigenvalues, the Schur form and the Schur vectors are computed.

Constraint: **job** = 'N', 'S' or 'V'.

2: **a**(**lda**,$*$) **– complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n general matrix A.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **n**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldz, rwork, work, lwork

5.4 Output Parameters

1: **a(lda,∗) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **job** = 'S' or 'V', **a** contains the upper triangular matrix T, the Schur form of A.

If **job** = 'N', the contents of **a** are overwritten.

2: **w(∗) – complex array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

The computed eigenvalues.

If **job** = 'S' or 'V', the eigenvalues occur in the same order as on the diagonal of T.

3: **z(ldz,∗) – complex array**

The first dimension, **ldz**, of the array **z** must satisfy

if **job** = 'N' or 'S', $\mathbf{ldz} \geq 1$;
if **job** = 'V', $\mathbf{ldz} \geq \max(1, \mathbf{n})$.

The second dimension of the array must be at least $\max(1, \mathbf{n})$ if **job** = 'V', and at least 1 otherwise

If **job** = 'V', **z** contains the unitary matrix Z of Schur vectors.

If **job** = 'N' or 'S', **z** is not referenced.

4: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **job** ≠ 'N', 'S' or 'V',
or **n** < 0,
or **lda** < $\max(1, \mathbf{n})$,
or **ldz** < 1, or **ldz** < **n** and **job** = 'V',
or **lwork** < $\max(1, 2 \times \mathbf{n})$.

ifail = 2

The QR algorithm failed to compute all the eigenvalues.

7 Accuracy

The computed Schur factorization is the exact factorization of a nearby matrix $A + E$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***.

If λ_i is an exact eigenvalue, and $\tilde{\lambda}_i$ is the corresponding computed value, then

$$\left|\tilde{\lambda}_i - \lambda_i\right| \leq \frac{c(n)\epsilon\|A\|_2}{s_i},$$

where $c(n)$ is a modestly increasing function of n, and s_i is the reciprocal condition number of λ_i. The condition numbers s_i may be computed by calling f08qy.

8 Further Comments

f02ga calls functions from LAPACK in Chapter F08. It first reduces A to upper Hessenberg form H, using a unitary similarity transformation: $A = QHQ^{\mathrm{H}}$. If only eigenvalues or the Schur form are required, the function uses the upper Hessenberg QR algorithm to compute the eigenvalues or Schur form of H. If the Schur vectors are required, the function first forms the unitary matrix Q that was used in the reduction to Hessenberg form; it then uses the QR algorithm to reduce H to T, using further unitary transformations: $H = STS^{\mathrm{H}}$; and at the same time it accumulates the matrix of Schur vectors $Z = QS$.

Each Schur vector z is normalized so that $\|z\|_2 = 1$, and the element of largest absolute value is real and positive.

The time taken by the function is approximately proportional to n^3.

9 Example

```
job = 'Vectors';
a = [complex(-3.97, -5.04), complex(-4.11, +3.7), complex(-0.34, +1.01),
complex(1.29, -0.86);
       complex(0.34, -1.5), complex(1.52, -0.43), complex(1.88, -5.38),
complex(3.36, +0.65);
       complex(3.31, -3.85), complex(2.5, +3.45), complex(0.88, -1.08),
complex(0.64, -1.48);
      complex(-1.1, +0.82), complex(1.81, -1.59), complex(3.25, +1.33),
complex(1.57, -3.44)];
[aOut, w, z, ifail] = f02ga(job, a)
```

```
aOut =
   -6.0004 - 6.9998i   -0.4701 - 0.2119i    0.0438 + 0.5124i   -0.9097 -
0.0925i
         0              -5.0000 + 2.0060i    0.7150 - 0.1028i   -0.0580 +
0.2575i
         0                    0              7.9982 - 0.9964i   -0.2232 -
1.0549i
         0                    0                   0             3.0023 -
3.9998i
w =
  -6.0004 - 6.9998i
  -5.0000 + 2.0060i
   7.9982 - 0.9964i
   3.0023 - 3.9998i
z =
    0.8457              -0.3613 + 0.1351i   -0.1755 + 0.2297i    0.1099 -
0.2007i
   -0.0177 + 0.3036i   -0.3366 + 0.4660i    0.7228               0.0336 +
0.2312i
    0.0875 + 0.3115i    0.6311               0.2871 + 0.4999i    0.0944 -
0.3947i
```

```
  -0.0561 - 0.2906i  -0.1045 - 0.3339i   0.2476 + 0.0195i   0.8534
ifail =
           0
```